

Sample Question Paper - 16
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes

Maximum Marks: 40

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

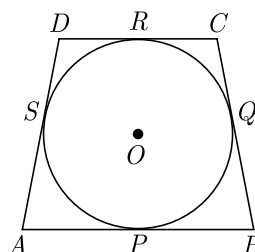
SECTION A

1. Which term of the AP 8, 14, 20, 26, will be 72 more than its 41st term.

OR

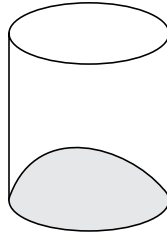
What is the n^{th} term of the AP $a, 3a, 5a, \dots$?

2. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$ then what is the angle of elevation of the sun ?
3. In Figure a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre O , in such a way that the sides AB , BC , CD , and DA touch the circle at the points P, Q, R and S respectively. Prove that. $AB + CD = BC + DA$.

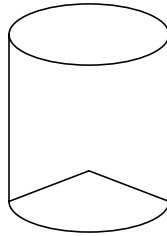


4. If a cone is cut into two parts by a horizontal plane passing through the mid-points of its axis, find the ratio of the volume of the upper part and the cone.
5. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice. Aisha, a juice seller, was serving juice to her customers in two types of glasses. Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.
First Type : A glass with hemispherical raised bottom.





Second Type : A glass with conical raised bottom of height 1.5 cm.



Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much?

6. Find the mean the following distribution :

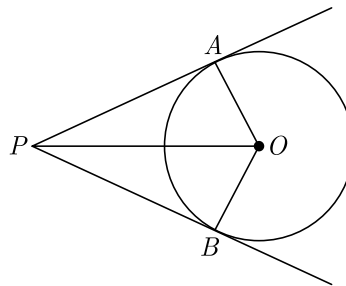
Class	3-5	5-7	7-9	9-11	11-13
Frequency	5	10	10	7	8

OR

The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then what is the median of the new set?

Section B

7. In the given figure, OP is equal to the diameter of a circle with centre O and PA and PB are tangents. Prove that ABP is an equilateral triangle.



8. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

9. If the median for the following frequency distribution is 28.5, find the value of x and y :

Class	Frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

10. Find the mean and mode of the following frequency distribution :

Classes	0- 10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	3	8	10	15	7	4	3

OR

The mean of n observations is \bar{x} , if the first term is increased by 1, second by 2 and so on. What will be the new mean ?

Section C

11. Find the number of terms of the AP $-12, -9, -6, \dots, 21$. If 1 is added to each term of this AP, then find the sum of all the terms of the AP thus obtained.
12. Draw two concentric circle of radii 3 cm and 5 cm. Taking a point on the outer circle, construct the pair of tangents to the inner circle.

OR

Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm, and taking B as centre draw another circle of radius 3 cm. Construct tangents to each circle of radius centre of the other circle.

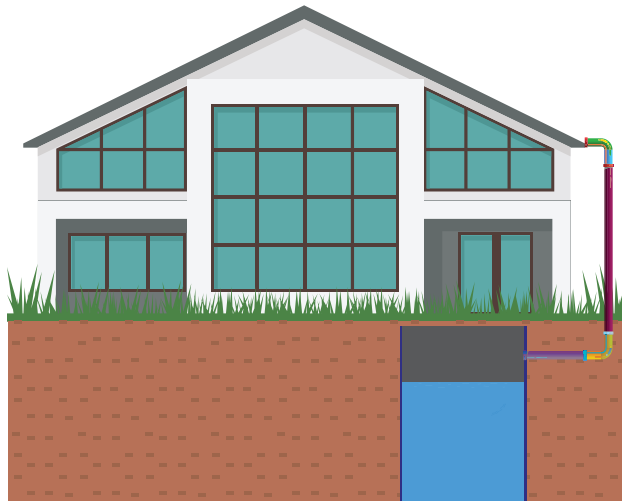
13. Supreme Court : The “Conference handshake” has been a tradition since the late 19th century. When the Justices of the Supreme Court assemble to go on the Bench each day, each Justice shakes hands with each of the other Justices for a total of 36 handshakes. The total number of handshakes h possible for n people is given by $h = \frac{1}{2}(n^2 - n)$.
- (i) Write an equation which represent above situation
- (ii) Solve above equation to determine the number of Justices on the Supreme Court.

Figure is given on next Page.

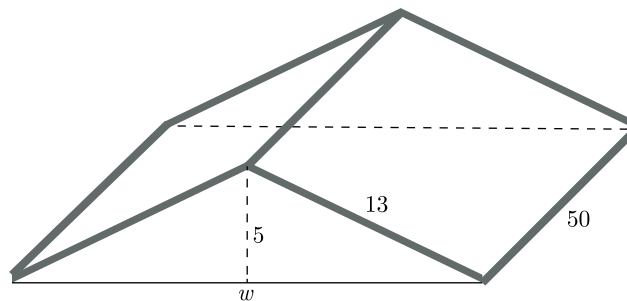




14. Rain Water Harvesting : Water harvesting is the activity of direct collection of rainwater, rather than allowing it to run off. The rainwater collected can be stored for direct use or can be recharged into the groundwater.



Rajkumar has installed rain water harvesting system in his house. He has a roof with dimensions shown below. He knows that the average rainfall in his suburb is 77 cm per year. Rajkumar would like to install a underground cylindrical rain water tank to hold the water that runs off the roof.



Can you help the Rajkumar answer the following questions

- On average what volume of water will fall on the roof each year ?
- If the tank has base diameter 7 m, how deep it need to be?

Solution
MATHEMATICS BASIC 241
Class 10 - Mathematics

Time Allowed: 120 minutes

Maximum Marks: 40

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
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5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. Which term of the AP 8, 14, 20, 26, will be 72 more than its 41st term.

Sol :

Let the first term be a , common difference be d and n th term be a_n .

We have $a = 8, d = 6$. Since n^{th} term is 72 more than 41st term. we get

$$a_n = a_{41} + 72$$

$$8 + (n-1)6 = 8 + 40 \times 6 + 72$$

$$6n - 6 = 240 + 72$$

$$6n = 312 + 6 = 318$$

$$n = 53$$

or

What is the n^{th} term of the AP $a, 3a, 5a, \dots$?

Sol :

Given AP is $a, 3a, 5a, \dots$

First term is a and $d = 3a - a = 2a$

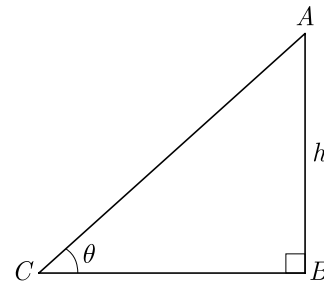
$$\begin{aligned} n^{\text{th}} \text{ term } a_n &= a + (n-1)d \\ &= a + (n-1)2a \\ &= a + 2na - 2a \\ &= 2na - a = (2n-1)a \end{aligned}$$

2. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$ then what is the angle of elevation of the sun ?

Sol :

Let AB be the rod of length h , BC be its shadow of

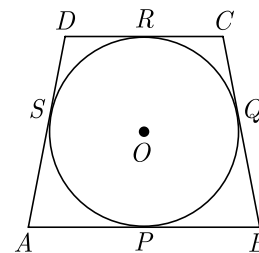
length $\sqrt{3}h$, θ be the angle of elevation of the sun.



In ΔABC , $\frac{h}{\sqrt{3}h} = \tan \theta$

$$\tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

3. In Figure a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre O , in such a way that the sides AB, BC, CD , and DA touch the circle at the points P, Q, R and S respectively. Prove that. $AB + CD = BC + DA$.



Sol :

Since length of tangents from an external point to a circle are equal,

At A , $AP = AS$... (1)

At B , $BP = BQ$... (2)

$$\text{At } C, \quad CR = CQ \quad \dots(3)$$

$$\text{At } D, \quad DR = DS \quad \dots(4)$$

Adding eqn. (1), (2), (3), (4)

$$AP + BP + DR + CR = AS + DS + BQ + CQ$$

$$AP + BP + DR + RC = AS + SD + BQ + QC$$

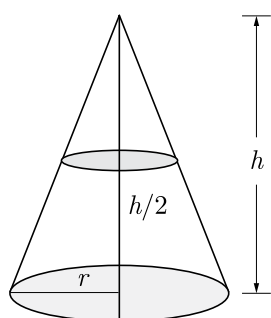
$$AB + CD = AD + BC$$

Hence Proved

4. If a cone is cut into two parts by a horizontal plane passing through the mid-points of its axis, find the ratio of the volume of the upper part and the cone.

Sol :

As per question the figure is shown below.



$$\begin{aligned} \text{Volume of upper cone} &= \frac{1}{3} \pi \left(\frac{r}{2}\right)^2 \times \frac{h}{2} \\ &= \frac{1}{24} \pi r^2 h \end{aligned}$$

$$\text{Volume of full cone} = \frac{1}{3} \pi r^2 h$$

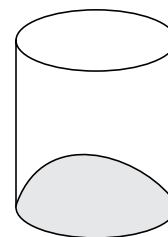
$$\begin{aligned} \frac{\text{Volume of upper of cone}}{\text{Volume of full cone}} &= \frac{\frac{1}{24} \pi r^2 h}{\frac{1}{3} \pi r^2 h} = \frac{1}{8} \\ &= 1 : 8 \end{aligned}$$

5. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice.

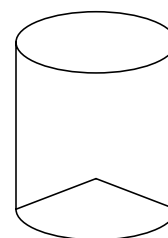
Aisha, a juice seller, was serving juice to her customers in two types of glasses.

Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.

First Type : A glass with hemispherical raised bottom.



Second Type : A glass with conical raised bottom of height 1.5 cm.



Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much?

Sol :

Let H and h be the height of cylinder and height of cone. Let r be the common radius of cone and cylinder and hemisphere.

Capacity of first glass,

$$\begin{aligned} &= \text{Volume of cylinder} - \text{Volume of hemisphere} \\ &= \pi r^2 H - \frac{2}{3} \pi r^3 \\ &= \pi r^2 \left[H - \frac{2}{3} r \right] \\ &= \pi \times (3)^2 \left[10 - \frac{2}{3} \times 3 \right] \\ &= 9\pi \times 8 = 72\pi \text{ cm}^2 \end{aligned}$$

Capacity of second glass,

$$\begin{aligned} &= \text{Volume of cylinder} - \text{Volume of cone} \\ &= \pi r^2 H - \frac{1}{3} \pi r^2 h = \pi r^2 \left[H - \frac{1}{3} h \right] \\ &= \pi (3)^2 \left[10 - \frac{1}{3} \times 1.5 \right] \\ &= 9\pi \times 9.5 = 85.5\pi \text{ cm}^2 \end{aligned}$$

Therefore Suresh got more juice of quantity,

$$= 85.5\pi - 72\pi \text{ cm}^2 = 13.5\pi \text{ cm}^3$$

6. Find the mean the following distribution :

Class	3-5	5-7	7-9	9-11	11-13
Frequency	5	10	10	7	8

Sol :

Class	Frequency (f_i)	Mid-Value (x_i)	$f_i x_i$
3-5	5	4	20
5-7	10	6	60
7-9	10	8	80
9-11	7	10	70
11-13	8	12	96
	$\sum f_i = 40$		$\sum f_i x_i = 326$

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{326}{40} = 8.15$

or

The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then what is the median of the new set?

Sol :

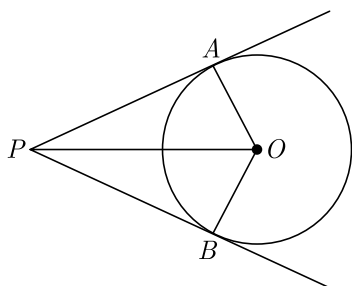
Since, $n = 9$

then, median term = $\left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}}$ item.

Now, last four observations are increased by 2, but median is 5^{th} observation, which is remaining unchanged. Hence there will be no change in median.

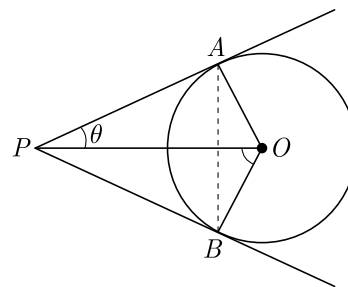
Section B

7. In the given figure, OP is equal to the diameter of a circle with centre O and PA and PB are tangents. Prove that ABP is an equilateral triangle.



Sol :

We redraw the given figure by joining A to B as shown below.



Since OA is radius and PA is tangent at A , $OA \perp AP$. Now in right angle triangle $\triangle OAP$, OP is equal to diameter of circle, thus

$$OP = 2OA$$

$$\frac{OA}{OP} = \frac{1}{2}$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Since PO bisect the angle $\angle APB$,

Hence, $\angle APB = 2 \times 30^\circ = 60^\circ$

Now, in $\triangle APB$,

$$AP = AB$$

$$\angle PAB = \angle PBA$$

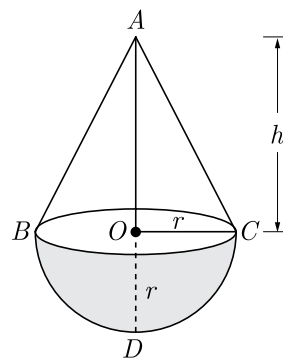
$$= \frac{180^\circ - 60^\circ}{2} = 60^\circ$$

Thus $\triangle APB$ is an equilateral triangle.

8. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

Sol :

Let ABC be a cone, which is mounted on a hemisphere.



We have $OC = OD = r$

Curved surface area of the hemispherical part

$$= \frac{1}{2}(4\pi r^2) = 2\pi r^2$$

Slant height of a cone, $l = \sqrt{r^2 + h^2}$

Curved surface area of a cone $= \pi r l = \pi r \sqrt{h^2 + r^2}$

Since curved surface areas of the hemispherical part and the conical part are equal,

$$2\pi r^2 = \pi r \sqrt{h^2 + r^2}$$

$$2r = \sqrt{h^2 + r^2}$$

Squaring both of the sides, we have

$$4r^2 = h^2 + r^2$$

$$4r^2 - r^2 = h^2$$

$$3r^2 = h^2$$

$$\frac{r^2}{h^2} = \frac{1}{3}$$

$$\frac{r}{h} = \frac{1}{\sqrt{3}}$$

Hence, the ratio of the radius and the height is $1 : \sqrt{3}$

9. If the median for the following frequency distribution is 28.5, find the value of x and y :

Class	Frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Sol :

We prepare following cumulative frequency table to find median class.

Class	f	$c.f.$
0-10	5	5
10-20	x	$x + 5$
20-30	20	$x + 25$
30-40	15	$x + 40$
40-50	y	$x + y + 40$
50-60	5	$x + y + 45$
	$\sum f = 60$	

Since, median is 28.5 which lies between 20-30. Thus model class is 20-30.

From table $N = x + y + 45$

$$60 = x + y + 45$$

$$x + y = 60 - 45 = 15 \quad \dots(1)$$

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f} \right) h$$

$$28.5 = 20 + \frac{[30 - (x + 5)]}{20} \times 10$$

$$8.5 = \frac{25 - x}{2}$$

$$25 - x = 17 \Rightarrow x = 25 - 17 = 8$$

$$\text{From (1), } y = 15 - 8 = 7$$

Hence, $x = 8$ and $y = 7$

10. Find the mean and mode of the following frequency distribution :

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	3	8	10	15	7	4	3

Sol :

We prepare following table to find mean.

Classes	x_i	f_i	$f_i x_i$
0-10	5	3	15
10-20	15	8	120
20-30	25	10	250
30-40	35	15	525
40-50	45	7	315
50-60	55	4	220
60-70	65	3	195
		$\sum f_i = 50$	$\sum f_i x_i = 1640$

$$\text{Mean } M = \frac{\sum f_i x_i}{\sum f_i} = \frac{1640}{50} = 32.8$$

Class 30-40 has the maximum frequency 35, therefore this is model class.

$$\text{Here } l = 30, f_1 = 15, f_2 = 7, f_0 = 10, h = 10$$

$$\text{Mode, } M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h$$

$$= 30 + \frac{15 - 10}{30 - 10 - 7} \times 10$$

$$= 30 + \frac{5}{13} \times 10$$

$$= 30 + \frac{50}{13} = 30 + 3.85 = 33.85$$



or

The mean of n observations is \bar{x} , if the first term is increased by 1, second by 2 and so on. What will be the new mean ?

Sol :

I term +1

II term +2

III term +3

n terms + n

The mean of the new numbers is

$$\bar{x} + \frac{\frac{n(n+1)}{2}}{n} = \bar{x} + \frac{(n+1)}{2}$$

Section C

11. Find the number of terms of the AP $-12, -9, -6, \dots, 21$. If 1 is added to each term of this AP, then find the sum of all the terms of the AP thus obtained.

Sol :

Let the first term be a , common difference be d , n th term be a_n and sum of n term be S_n

We have $a = -12, d = -9 - (-12) = 3$

$$a_n = a + (n-1)d$$

$$21 = -12 + (n-1) \times 3$$

$$21 + 12 = (n-1) \times 3$$

$$33 = (n-1) \times 3$$

$$n-1 = 11$$

$$n = 11 + 1 = 12$$

Now, if 1 is added to each term we have a new AP with $-12 + 1, -9 + 1, -6 + 1, \dots, 21 + 1$

Now we have $a = -11, d = 3$ and $a_n = 22$ and $n = 12$

Sum of this obtained AP,

$$S_{12} = \frac{12}{2}[-11 + 22] = 6 \times 11 = 66$$

Hence the sum of new AP is 66.

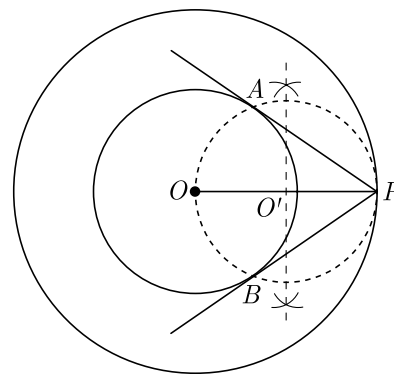
12. Draw two concentric circle of radii 3 cm and 5 cm. Taking a point on the outer circle, construct the pair of tangents to the inner circle.

Sol :

Steps of Construction :

1. Draw a circle with radius 3 cm and centre O .

2. Draw another circle with centre O and radius 5 cm.
3. Take a point P on the circumference of outer circle and join O to P .
4. Taking OP as diameter draw another circle which intersect the smallest circle at A and B .
5. Join A to P and B to P . AP and BP are the required tangents.



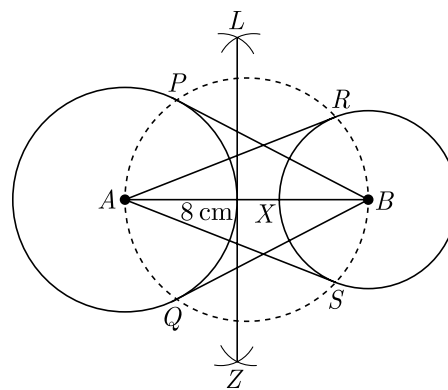
or

Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm, and taking B as centre draw another circle of radius 3 cm. Construct tangents to each circle of radius centre of the other circle.

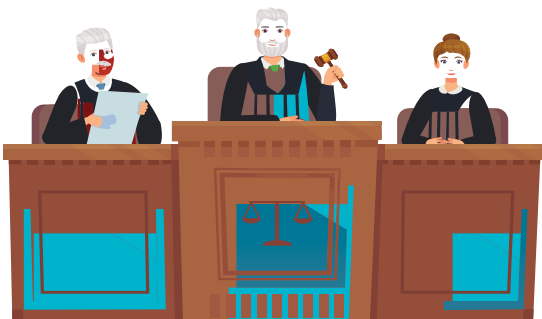
Sol :

Steps of Construction :

1. Draw a line segment AB of length 8 cm.
2. Draw a circle with centre A and radius 4 cm.
3. Draw another circle with centre B and radius 3 cm.
4. Taking AB as diameter draw another circle, which intersects first two circles at P and Q , and R and S .
5. Join B to P , B to Q , A to R and A to S . Thus BP, BQ, AR and AS are the required tangents.



13. Supreme Court : The “Conference handshake” has been a tradition since the late 19th century. When the Justices of the Supreme Court assemble to go on the Bench each day, each Justice shakes hands with each of the other Justices for a total of 36 handshakes. The total number of handshakes h possible for n people is given by $h = \frac{1}{2}(n^2 - n)$.
- Write an equation which represent above situation
 - Solve above equation to determine the number of Justices on the Supreme Court.



Sol :

- Since, $h = 36$ we have

$$36 = \frac{1}{2}(n^2 - n)$$

$$72 = n^2 - n$$

$$n^2 - n - 72 = 0$$

Thus, $n^2 - n - 72 = 0$ represent the given situation.

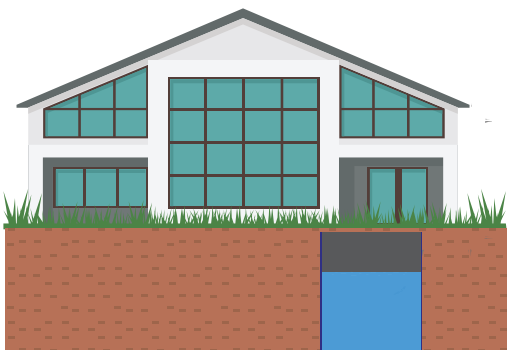
- $n^2 - 9n + 8n - 72 = 0$

$$n(n - 9) + 8(n - 9) = 0$$

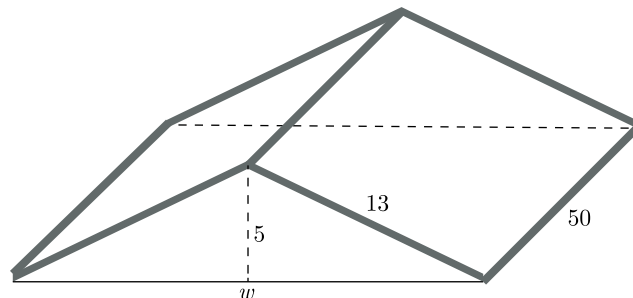
$$(n - 9)(n + 8) = 0 \Rightarrow n = 9, -8$$

There are 9 Justices on the Supreme Court.

14. Rain Water Harvesting : Water harvesting is the activity of direct collection of rainwater, rather than allowing it to run off. The rainwater collected can be stored for direct use or can be recharged into the groundwater.



Rajkumar has installed rain water harvesting system in his house. His has a roof with dimensions shown. below. He knows that the average rainfall in his suburb is 77 cm per year. Rajkumar would like to install a underground cylindrical rain water tank to hold the water that runs off the roof.



Can you help the Rajkumar answer the following questions

- On average what volume of water will fall on the roof each year ?
- If the tank has base diameter 7 m, how deep it need to be?

Sol :

Length of roof, $l = 50$ m

- For the rainfall calculation base area of roof will be taken. Let w be the width of base.

$$\text{Then, } \frac{w}{2} = \sqrt{13^2 - 5^2} = 12$$

$$w = 24 \text{ m}$$

Area of base, $S = w \times l$

$$= 24 \times 50 = 1200 \text{ m}^2$$

Since rainfall is 77 cm, the volume of water collected in a year,

$$V = 1200 \times 0.77 = 924 \text{ m}^3$$

- Volume of tank must be 924 m^3 . Let h be height of tank.

$$\text{Thus, } \pi r^2 h = 924$$

$$\frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times h = 924$$

$$h = \frac{924 \times 7 \times 2 \times 2}{22 \times 7 \times 7} = 24 \text{ m}$$

